Math 2201

Review Chapter One

1. Examine the following number patterns:

$$1^3 = 1 \quad and\ 1 = 1^2$$
$$1^3 + 2^3 = 9 \quad and \quad 9 = 3^2$$
$$1^3 + 2^3 + 3^3 = 36 \quad and \quad 36 = 6^2$$
$$1^3 + 2^3 + 3^3 + 4^3 = 100 \quad and\ 100 = 10^2$$

A) Describe the pattern you see. The sum of the digits squared equals the sum of the cubes

B) Use your observation to predict the next equation in the pattern. $1^3 + 2^3 + 3^3 + 4^3 + 5^3 = (1 + 2 + 3 + 4 + 5)^2 = 15^2 = 225$

C) Make a conjecture about the sum of the first n cubes. The sum of the first n cubes equals the square of the sum of the first n numbers.

2. Sadie claims that the difference between any two positive integers is always a positive integer. Do you agree or disagree? Use inductive reasoning to justify your answer. (12)-(16)=- 4

3. Prove, using deductive reasoning, that the product of two odd integers is always odd.
$(2x + 1)(2x + 3) = 4x^2 + 8x + 3$ Since $4x^2\ and\ 8x$ are always even – adding 3 will always make it odd!

4. Examine this pattern to determine the next equation.

37 x 3 = 111
37 x 6 = 222
37 x 9 = 333
37 x 12 = 444 37x15 = 555

Is your conjecture correct? Explain how you know.

The next multiple of 3 is 15 and 37 x 15 is 555!

5. Frank tosses a coin five times , each time it comes up tails. He makes the following conjecture: The coin will come up tails on every toss. Is his conjecture reasonable? Explain.

No – there is always a 50/50 chance that each new toss will be heads or tails!

6. Prove, deductively, that the product of two consecutive odd integers is always odd.

Same as 3

7. The following proof seems to show that 10 =9.9999….. Is this proof valid? Explain

Let a = 9.99999….

10a = 99.99999…	Multiply by 10	
10a – a = 90	Subtract a	No subtract a or 9.999... from both sides
9a = 90	Simplify	9a = 99.9999... – a
a = 10	Divide by 9	

8. Julie was trying to prove that a number trick always results in 5:

N	Choose a number	
N+10	Add 10	
5N + 10	Multiply the total by 5	5N + 50
5N - 40	Subtract 50	5N
$\frac{5N-40}{N}$	Divide by the number you started with. 5	

Identify the error in Julie's proof, and correct it.

9. Andy , Bonnie, Candice, and Darlene are standing in line to buy ice cream. Determine the order in which they are lined up, using these clues:
   - Candice is between Andy and Bonnie
   - Darlene is next to Andy
   - Bonnie is not first

	1	2	3	4
Andy		y		
Bonnie	x			y
Candice			y	
Darlene	y			

Darlene, Andy, Candice, Bonnie

10. Two mothers and a daughter got off a city bus, reducing the number of passengers by three. Explain how this is possible. Mother – her daughter-the daughter's daughter – 3 people

11. Three little pigs built three houses: one of straw , one of sticks, and one of bricks. By reading the six clues, deduce which pig built each house, and the town in which it was located.
    Clues
    - Penny Pig did not build a brick house
    - The straw house was not medium In size
    - Perry Pig's house was made of sticks, and it was neither medium nor small in size
    - Patricia Pig built her house in Marystown
    - The house in Lawn was large
    - One house was in a town called Epworth

    Penny – straw, Epworth, small
    Perry – sticks, Lawn, large
    Patricia – brick, Marystown, medium

12. Prove the following trick always ends in 10. Do one example and then use deductive reasoning.

• Choose a natural number	10	n
• Double it	2x10=20	2n
• Add 20	20+20 = 40	2n+20
• Divide by 2	40/2 = 20	n+10
• Subtract the original number	20- 10 = 10	n+10-n = 10