Section 4.4: Simplifying Algebraic Expressions Involving Radicals

When a radical contains a variable, we must consider RESTRICTIONS on the variable.

Are there any values that ***x*** is not allowed to be?

Example 1: Identify the restrictions for each of the following.

a). $\sqrt{x}$

NOTE:
You cannot take the square root of a negative number!

b). $\sqrt{x-3}$

c). $\sqrt{x+2}$

Since you can't take the square root of a negative number,what's under the square root symbol has to be GREATER THAN or EQUAL TO ZERO.

Consider these. What are the restrictions now?

$$d)\ \frac{1}{\sqrt{x}}$$

$$e)\ \frac{1}{\sqrt{x-3}}$$

NOTE:
You cannot take the square root of a negative number OR have zero in the denominator.

$$f)\ \frac{1}{\sqrt{x+2}}$$

The restriction is GREATER THAN ZERO!

Recall: Exponent Law: When ***multiplying*** two powers that have the same base, we **ADD** the exponents.

$$x^2 \cdot x^2 =$$
$$x^3 \cdot x^3 =$$
$$x^4 \cdot x^4 =$$
$$x^5 \cdot x^5 =$$

This helps to explain how each square root below is simplified.

$$\sqrt{x^4} =$$
$$\sqrt{x^6} =$$
$$\sqrt{x^8} =$$
$$\sqrt{x^{10}} =$$

NOTE:
The exponent on each square root is one half the exponent on the radicand.

Even and Odd Exponents

Even Exponents	Odd Exponents
$\sqrt{x^{even}}$	$\sqrt{x^{odd}}$
• NO restrictions on variable, since $\sqrt{(+)^{even}} = \sqrt{+}$ and $\sqrt{(-)^{even}} = \sqrt{+}$	• Restriction: $x \geq 0$
• We write: $x \in R$	• We rewrite the radicand using: $x^{even} \bullet x$ therefore, $\sqrt{x}$ always remains.
• Simplify: $\sqrt{x^{12}}$	• Simplify: $\sqrt{x^7}$

Example 2: Simplify and identify the restriction on each variable.

a). $\sqrt{x^{13}}$

b). $\sqrt{y^{25}}$

c). $\sqrt{9x^{11}}$

d). $\sqrt{12x^7y^{16}}$

e). $2\sqrt{28x^{13}}$

f). $\sqrt[3]{8y^6x^{10}}$

Operations with Radicals Containing Variables:

Adding/Subtracting

→ We still need LIKE radicals to add or subtract.

Example 3: Simplify first, then combine like radicals.

a). $3\sqrt{x} + 5\sqrt{x} - \sqrt{x}$

b). $\sqrt[3]{27x^7} + 2\sqrt[3]{8x^4}$

c). $5\sqrt{x^6} + 2\sqrt{y^4} - \sqrt{x^6} + 4\sqrt{y^4}$

What are the restrictions on each variable above?

Your Turn:

d). $4\sqrt{x} + 2\sqrt{x} - 7\sqrt{x}$

e). $6\sqrt{32x^3} - 5\sqrt{8x^3} + 3\sqrt{2x^3}$

Multiplying:

↳ Remember $c\sqrt{a} \cdot d\sqrt{b} = c \cdot d\sqrt{a \cdot b}$

Distributive Property and FOIL

Example 4: Simplify and identify the restrictions on the variable.

a). $\left(5\sqrt{x}\right)\left(-4\sqrt{x^3}\right)$

b). $\left(2\sqrt{x}+1\right)\left(3-6\sqrt{x}\right)$

Dividing:

↳ Remember $\dfrac{c\sqrt{a}}{d\sqrt{b}} = \dfrac{c}{d}\sqrt{\dfrac{a}{b}}$

Rationalize the denominator if necessary

Example 5: Combine into a single radical where possible and simplify.

a). $\dfrac{\sqrt{10x^9}}{\sqrt{5x^3}}$

b). $\dfrac{\sqrt{2}}{\sqrt{5x}}$

c). $\dfrac{10}{3\sqrt{x^3}}$

d). $\dfrac{4+2\sqrt{x}}{\sqrt{x}}$

Assign: p. 211 - 213, # 1-6, 8-12