

1. Given the standard form of the quadratic function,  $y = -\frac{1}{2}x^2 + 4x - 3$ :

- a) Will the graph be wider or narrower than the original  $y = x^2$ ?  $a = -\frac{1}{2}$  wider
- b) What is the direction of opening? down, since  $a$  is negative
- c) What are the coordinates of the vertex?

$$x = \frac{-b}{2a} = \frac{-4}{2(-\frac{1}{2})} = \frac{-4}{-1} = 4 \quad y = -\frac{1}{2}(4)^2 + 4(4) - 3 = -8 + 16 - 3 = 5 \quad \text{vertex } (4, 5)$$

d) How many x-intercepts will the graph have? 2 x-ints. ooo



2. Given the quadratic function  $y = \frac{1}{2}(x+7)(x-3)$ , determine the

a) x-intercepts:  $x = -7$   $(-7, 0)$   
 $x = 3$   $(3, 0)$

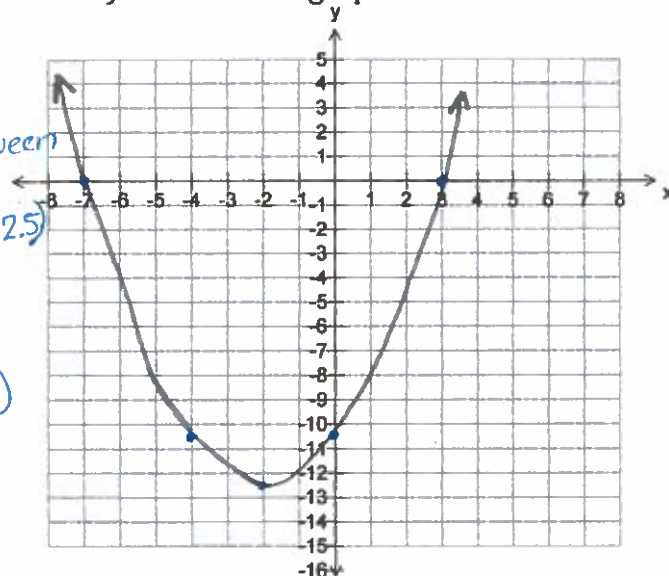
b) coordinates of the vertex: halfway between two x-int.  
 $x = \frac{-7+3}{2} = \frac{-4}{2} = -2$   $(-2, -12.5)$

$$y = 0.5(-2+7)(-2-3) = 0.5(5)(-5) = -12.5$$

c) y-intercept: when  $x = 0$   
 $y = 0.5(7)(-3) = -10.5$   $(0, -10.5)$

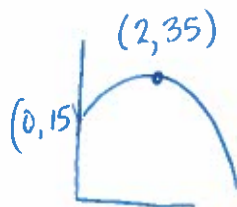
d) range:  
 $y \geq -12.5$

e) sketch the graph



3. A model rocket is launched from its launch pad which is 15 m above the ground. It takes 2 seconds for the rocket to reach a maximum height of 35 m.

a) Algebraically determine the quadratic function in the form  $y = a(x-h)^2 + k$ , that models the path followed by the rocket.



$$y = a(x-2)^2 + 35$$

$$15 = a(-2)^2 + 35$$

$$15 = 4a + 35$$

$$-20 = 4a$$

$$a = -5$$

$$y = -5(x-2)^2 + 35$$

b) Determine the height of the rocket at 3.5 s.

$$y = -5(x-2)^2 + 35 \quad y = -5(2.25) + 35$$

$$y = -5(3.5-2)^2 + 35 \quad y = -11.25 + 35$$

$$y = -5(1.5)^2 + 35 \quad y = 23.75 \text{ m}$$

4. Larry operates a popular hot dog stand. He sells 550 hot dogs per day at \$2 each. The previous year's sales show that for every \$0.50 increase in price, he will sell 50 fewer hot dogs.

a) Write a quadratic equation that models this problem.

Revenue = (number sold)(cost)  
 Let  $x$  = number of \$0.50 increases

$$R(x) = (550 - 50x)(2 + 0.50x)$$

$$= 1100 + 275x - 100x - 25x^2$$

$$= -25x^2 + 175x + 1100$$

$$\left. \begin{aligned} x &= \frac{-b}{2a} = \frac{-175}{2(-25)} \\ &= \frac{-175}{-50} = 3.5 \end{aligned} \right\}$$

b) What price should Larry charge to maximize his revenue?

$$\text{Cost} = 2 + 0.5x = 2 + 0.5(3.5) = 2 + 1.75 = \$3.75$$

c) What is the maximum revenue?

$$\begin{aligned} R(3.75) &= -25(3.5)^2 + 175(3.5) + 1100 \\ &= -25(12.25) + 612.5 + 1100 = -306.25 + 1712.50 \\ &= \$1406.25 \end{aligned}$$

5. A ball is thrown into the air and its height  $h(t)$  above the ground, in meters, after  $t$  seconds is modeled by the function  $h(t) = -5t^2 + 20t + 2$ .

when  $t=0$

a) What is the initial height of the ball?

$$h(0) = -5(0)^2 + 20(0) + 2 = 2 \text{ m}$$

b) When did the ball reach its maximum height? find  $x$ -coordinate of vertex

$$x = \frac{-b}{2a} = \frac{-20}{2(-5)} = \frac{-20}{-10} = 2 \text{ sec.}$$

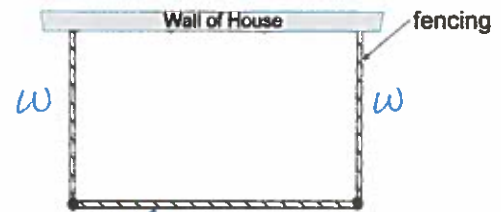
c) What was the maximum height of the ball? find  $y$ -coordinate of vertex

$$\begin{aligned} h(2) &= -5(2)^2 + 20(2) + 2 \\ &= -5(4) + 40 + 2 \\ &= -20 + 40 + 2 = 22 \text{ m} \end{aligned}$$

6. A rectangular region is to be constructed using 120m of fencing and a house as one side.

a) Write the quadratic function that models the area of the rectangular region.

$$\begin{aligned} A &= l \times w \\ &= (120 - 2w)w \\ &= 120w - 2w^2 \\ A &= -2w^2 + 120w \end{aligned}$$



b) Determine the width which maximizes the area.

$$w = \frac{-b}{2a} = \frac{-120}{2(-2)} = \frac{-120}{-4} = 30 \text{ m}$$

$$\begin{aligned} P &= 2w + l \\ 120 &= 2w + l \\ l &= 120 - 2w \end{aligned}$$

c) Determine the maximum enclosed area.

$$\begin{aligned} A &= -2(30)^2 + 120(30) \\ &= -2(900) + 3600 \\ &= -1800 + 3600 = 1800 \text{ m}^2 \end{aligned}$$

7. Given the function  $y = 2(x-3)^2 - 8$ , determine the following information and sketch the graph.

a) equation of the axis of symmetry:  $x = 3$

b) coordinates of vertex:  $(3, -8)$

c)  $y$ -intercept: when  $x = 0$

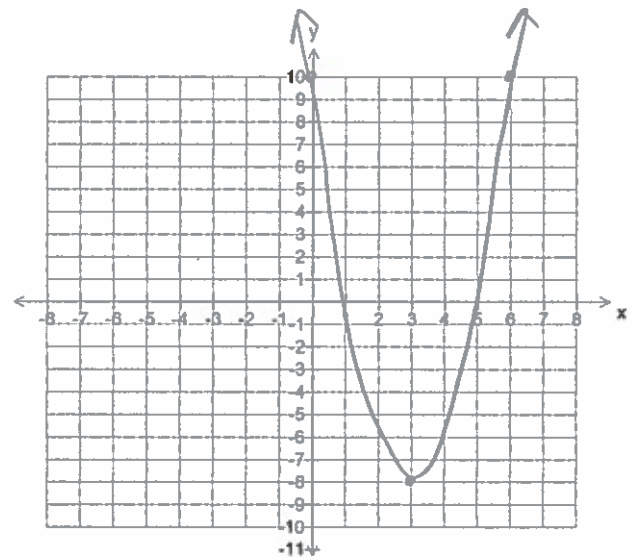
$$\begin{aligned} y &= 2(0-3)^2 - 8 \\ &= 2(-3)^2 - 8 \\ &= 2(9) - 8 \\ &= 18 - 8 = 10 \end{aligned} \quad (0, 10)$$

d) domain:

$$x \in \mathbb{R}$$

e) range:  $y \geq -8$

f) sketch:



g) What is the function in standard form?

$$\begin{aligned} y &= 2(x-3)(x-3) - 8 \\ y &= 2(x^2 - 6x + 9) - 8 \\ y &= 2x^2 - 12x + 18 - 8 \\ y &= 2x^2 - 12x + 10 \end{aligned}$$