Section 6.3: Factored Form of a Quadratic Function

make the connection between the factored form of a quadratic and the x-intercepts of the graph

Forms of a Quadratic Function

(i) Standard Form $y = ax^2 + bx + c$

(ii) Factored Form $y = a(x-r)(x-s)$

Example:

Factored Form $y = (x-2)(x+3)$

Standard Form $y = x^2 + x - 6$

Example:

Factored Form $y = 3(x-1)(x+5)$

Standard Form

x-intercepts from a Quadratic Function

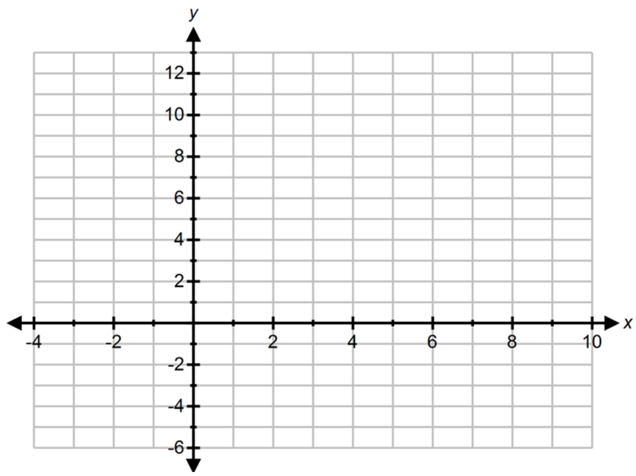
Investigate how to algebraically attain x – intercepts from a quadratic function in standard form $y = ax^2 + bx + c$.

Graph each of the quadratic functions:

https://www.desmos.com/

(a) $y=(x-2)(x-6)$

What is the standard form of the quadratic function?



co-ordinates of vertex:____

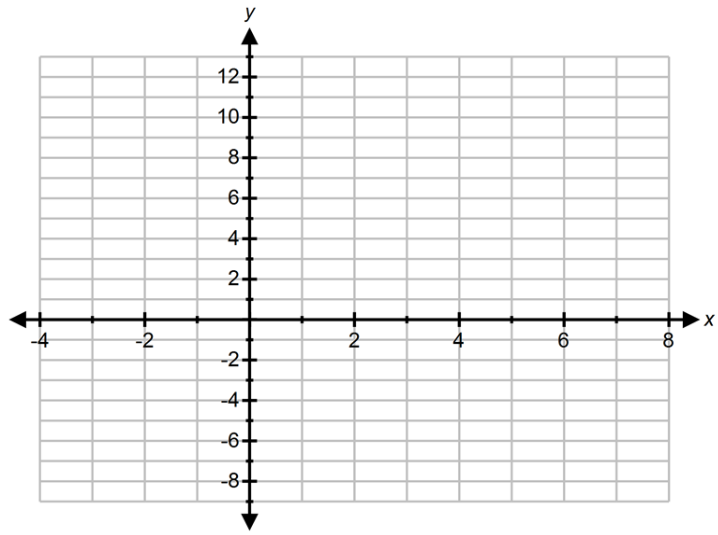
axis of symmetry:_____

co-ordinates of y – intercept:_____

co-ordinates of x – intercepts:__________

(b) $y = 2(x+1)(x-3)$

What is the standard form of the quadratic function?



co-ordinates of vertex:____

axis of symmetry:_____

co-ordinates of y – intercept:_____

co-ordinates of x – intercepts:__________

Using the results of the quadratic functions above and their respective graphs, answer each of the questions.

Forms of a Quadratic Function

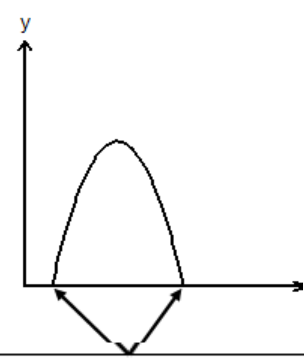
(i) Standard Form $y = ax^2 + bx + c$

(ii) Factored Form $y = a(x - r)(x - s)$

1. Which form of the quadratic function is easiest for determining the x – intercepts without the graph?

2. What is the connection between the factored form of a quadratic function and the x – intercepts?

Example: $y = (x - 2)(x - 6)$



Zeros of a Quadratic Function

- The zero(s) of a quadratic function represent the position(s) where the height is __________

- The zero(s) of a quadratic function are also referred to as the __________________

Algebraically Determine the x – intercepts of a Quadratic Function in Factored Form

$$y = a(x - r)(x - s)$$

the x-intercepts of the graph of a function is the same as the roots of the equation or the zeros of a function

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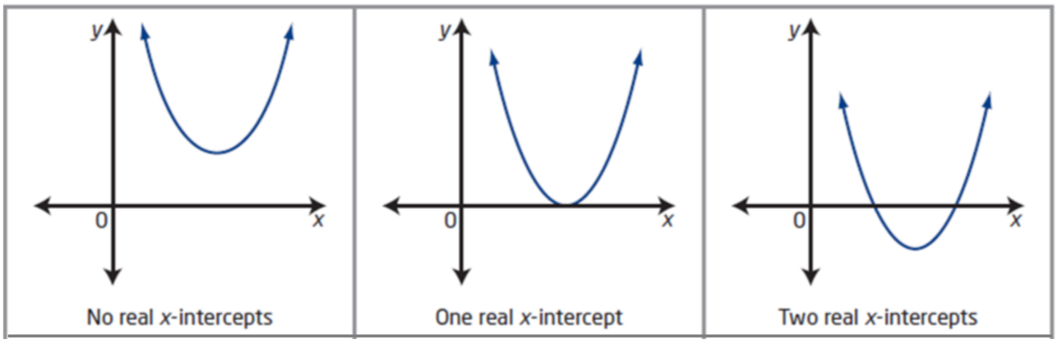
Possible Number of x-intercepts for a Quadratic Graph



x – intercepts of a Quadratic Function

Zero Product Property

If the product of two real numbers is zero ($a \cdot b = 0$)

then one or both must be zero.

In other words: $a = 0$ and $b = 0$

Examples:

$$y = x^2 + 6x + 5 \longrightarrow y = (x+5)(x+1)$$

$$y = x^2 - 2x - 8 \longrightarrow y = (x-4)(x+2)$$

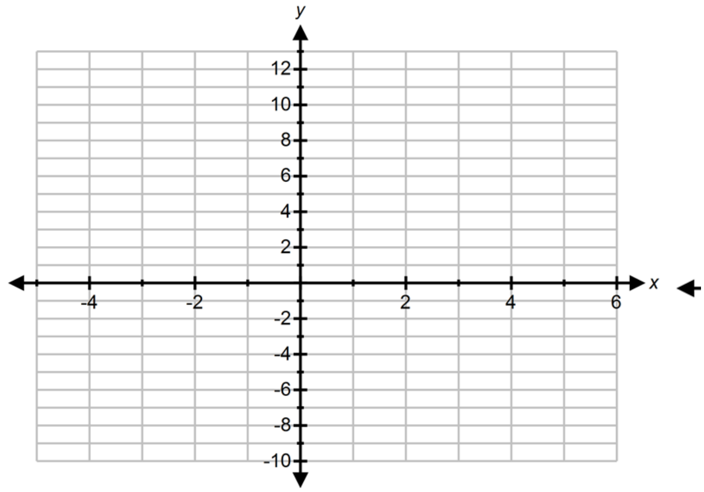
$$y = x^2 - x - 12 \longrightarrow y = (x-4)(x+3)$$

$$y = 2x^2 + 4x - 6 \longrightarrow y = 2(x-1)(x+3)$$

$$y = -x^2 - 3x - 2 \longrightarrow y = -(x+1)(x+2)$$

Example 1: Given the quadratic function $y = -(x+2)(x-4)$

(a) determine the x – intercepts



(b) determine the axis of symmetry

(c) determine the coordinates of the vertex

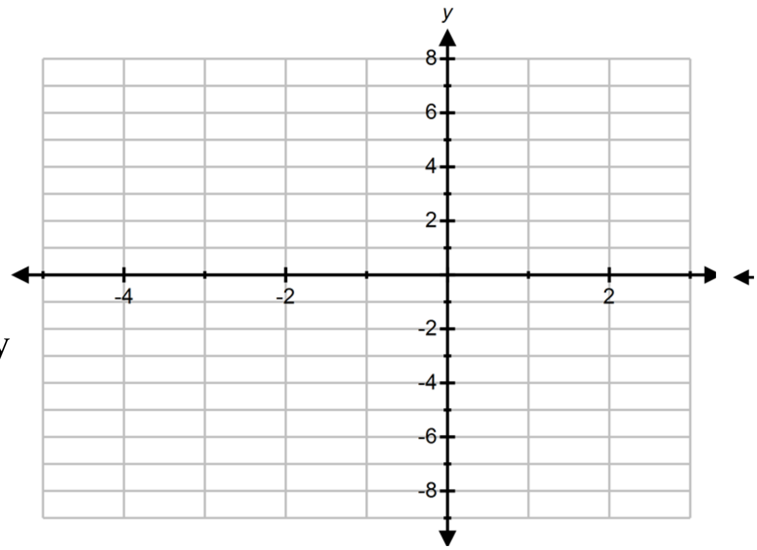
(d) determine the y – intercept

(e) sketch the graph

(f) state the range

Example 2: Given the quadratic function $y = 2x(x + 4)$

(a) determine the x – intercepts



(b) determine the axis of symmetry

(c) determine the coordinates of the vertex

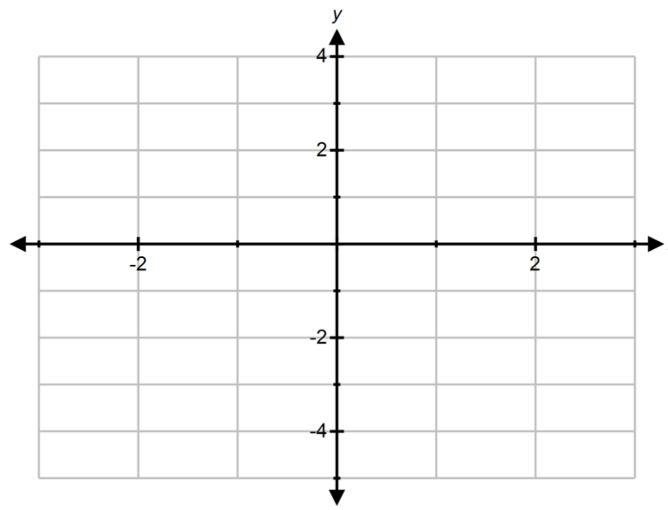
(d) determine the y – intercept

(e) sketch the graph

(f) state the range

Example 3: Given the quadratic function $y = -3(x-1)(x-1)$

(a) determine the x – intercepts



(b) determine the axis of symmetry

(c) determine the coordinates of the vertex

(d) determine the y – intercept

(e) sketch the graph

(f) state the range

Determine the Equation of a Quadratic Function (Standard Form)

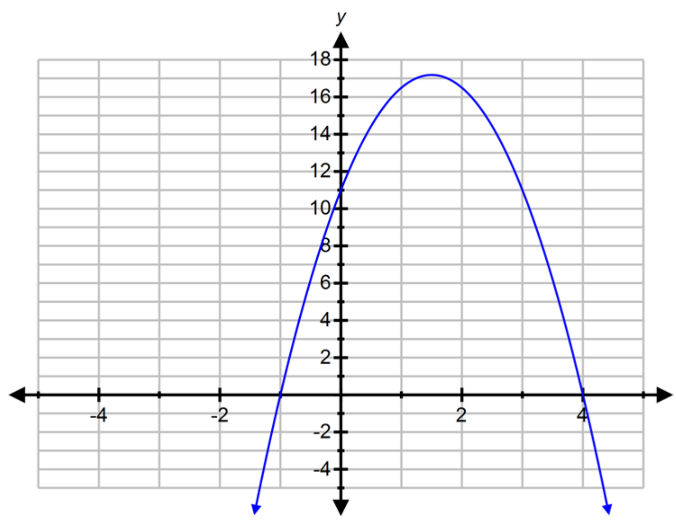
write the equation in factored form and then expand to standard form

$y = a(x - r)(x - s)$ $\longrightarrow$ $y = ax^2 + bx + c$

determine the x-intercepts and the a value.

Example 1: Determine the function that defines this parabola.

Express the function in standard form.



Step 1: Using the x-intercepts express the function in factored form

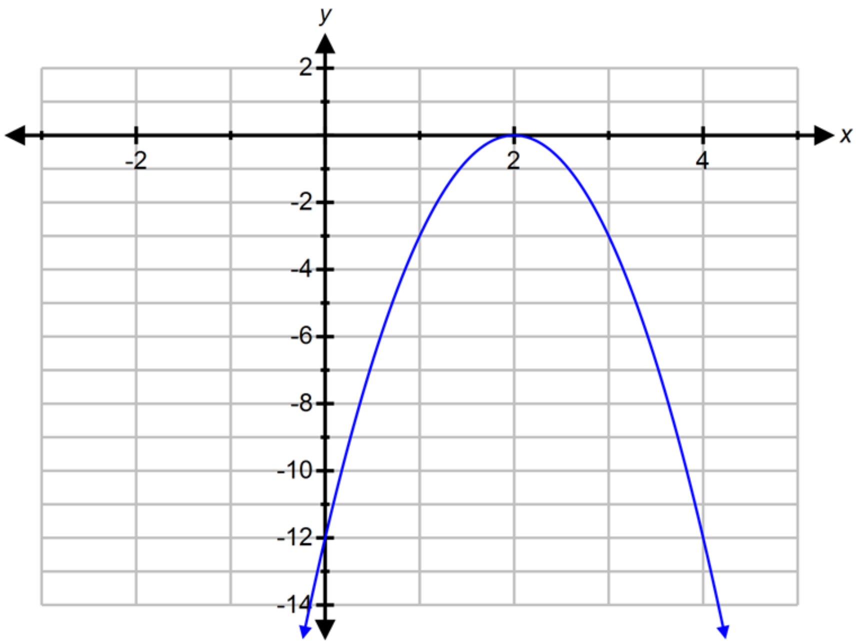
Step 2: Use the coordinates of another point to solve for the value of a

Step 3: Expand factored form to produce standard form

Example 2:

Determine the function that defines this parabola.

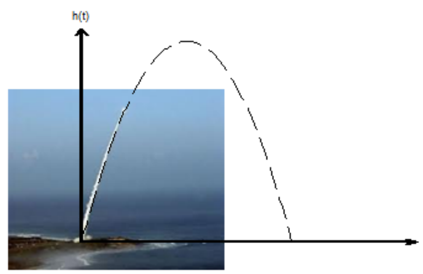
Express the function in standard form.



Determining the Equation of a Quadratic Function Based on a Verbal Description.

Example:

A missile fired from ground level attains a height of 180 m at 2 seconds. The missile is in the air for 6 seconds



(a) Determine the quadratic function that models the height of the missile over time.

(b) State the domain and range of the variables.

Domain:__________________

Range:___________________

Maximum/Minimum Word Problems

to solve max/min problems you will have to determine the highest (or lowest) point , in other words, **the vertex**.

Types:

1) Quadratic Equation is Given

2) Create a Quadratic Equation

- Area
- Revenue

(A) Determining the maximum height given the quadratic function.

Example 1:

A boat in distress fires off a flare. The height of the flare, h, in metres above the water, t seconds after shooting, is modeled by the function

$h(t) = -4.9t^2 + 29.4t + 3$.

Algebraically determine the maximum height attained by the flare.

Example 2:

The path of a volleyball is given by $h(t) = -\frac{1}{2}t^2 + 5t + 2$ where t is the time in seconds and h is the height in meters. At what time does the ball reach its maximum height?

Please note we do not have to go further to substitute it back into the equation to find the height. This question asks just when it happens.

Be very careful to INTREPRET the question correctly!

(B) Area Problems

(i) Open Field

Example 1:

A farmer is constructing a rectangular fence in an open field to contain cows. There is 120 m of fencing.

(a) Write the quadratic function that models the region.

(b) Determine the maximum area of the enclosed region.

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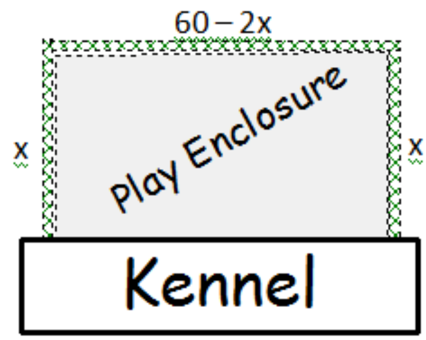
(i) Using a Physical Structure as One Side

Example 2:

A rectangular play enclosure for some dogs is to be made with 60 m of fencing using the kennel as one side of the enclosure as shown.

(a) Algebraically determine the quadratic function that models the area.



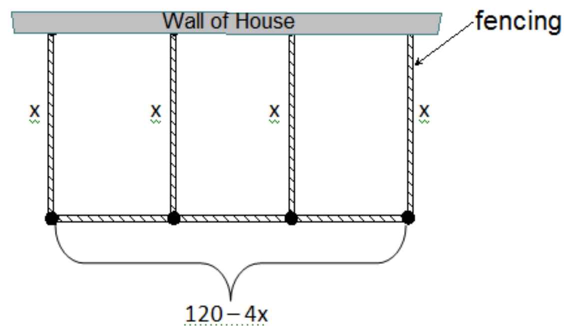
(b) Use the function to determine the maximum area.

(c) State the domain and range of the variables in the function.

Example 3:

A rectangular region, placed against the wall of a house, is divided into three regions of equal area using a total of 120 m of fencing as shown.

(a) Algebraically develop a quadratic function that models the area.



(b) Determine the maximum area of the pen.

(c) State the domain and range of the variables in the function.

(B) Revenue Problems

Formula for Revenue:
Revenue = (Number Sold)(Cost)

Example 1:

Global Gym charges its adult members $50 monthly for a membership. The club has 600 adult members. Global Gym estimates that for each $5 increment in the monthly fee, it will lose 50 members.

(a) Determine the function that models Global Gym's revenue.

(b) Determine the maximum revenue generated.

(c) Determine the monthly fee that will produce the greatest revenue.

Example 2:

An orange grower has 400 crates of oranges ready for market and will have 20 more crates each day that shipment is delayed. The present price is \$60 per crate however, for each day shipment is delayed, the price per crate decreases by \$2.

(a) Determine the revenue function that models this function.

(b) Determine the maximum revenue that can be generated.

(c) Determine the price per crate that will produce the greatest revenue.

Your Turn

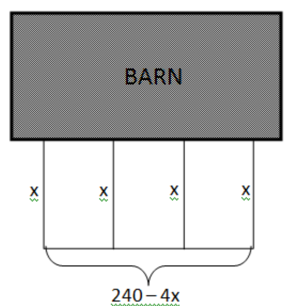
1. A dinner theatre show which sells out each night with 400 tickets currently cost \$10 per ticket. Proposed increases in ticket prices reveal that for each \$2 increase, 20 less people will attend. Write a quadratic function to model the theatre's revenue and determine the maximum revenue generated.

2. An Airline company sells 500 tickets per flight at a cost of \$100 per ticket. Proposed increases in ticket prices reveal that for each \$5 increase, 20 less people will purchase tickets. Write a quadratic function to model the Airline's revenue per flight and use it to determine the maximum revenue that can be generated per flight. What ticket price should the airline charge to maximize revenue?

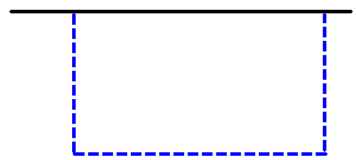
3. A barn which contains different livestock will use 240 m of fencing to construct three equal rectangular regions. There is no fencing along the side of the barn so livestock can move in and out of the barn. The quadratic function $A(x) = -4x^2 + 240x$ models the area of the pen where $A(x)$ represents the maximum area and x represents the width.

(a) Determine the maximum area of the pen.

(b) State the domain and range.



5. A lifeguard marks off a rectangular swimming area at a beach with 200 m of rope using the beach as one side. Determine the maximum area and the dimensions of the swimming area?



6. A farmer is going to construct a rectangular fence in an open field using 400m of fencing. Develop an appropriate quadratic function and use it to determine the maximum enclosed area and the dimensions of the rectangular region.

7. A rectangular storage area for heavy equipment is to be constructed using 148 m of fencing and a building as one side.

   Set up an appropriate equation and use it to determine the dimensions required to maximize the area enclosed.

Work Sample 6.3: pg.346 #s 1a-f, 2a-b, 4 ade, 5, 8a-b, 9, 11 a)c), 13, 15a-b